NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAMS	**LEVEL:** 7
COURSE CODE: NUM702S	**COURSE NAME:** NUMERICAL METHODS 2
SESSION: JANUARY 2020	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 90

SECOND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 4 to 5 decimals where necessary unless specified otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

Problem 1 [20 Marks]

1-1. Find the Padé approximation $R_{2,2}(x)$ for $f(x) = \ln(1+x)/x$ starting with the MacLaurin expansion

$$f(x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^4}{5} - \cdots. \qquad [12]$$

1-2. Use the result in **1-1.** to establish $\ln(1+x) \approx R_{3,2} = \dfrac{30x + 21x^2 + x^3}{30 + 36x + 9x^2}$ and express $R_{3,2}$ in continued fraction form. [8]

Problem 2 [25 Marks]

For any non negative interger n we define Chebyshev polynomial of the first kind as

$$T_n(x) = \cos(n\theta), \quad \text{where } \theta = \arccos(x), \text{ for } x \in [-1, 1].$$

2-1. Show the following property: [5]

$$T_n \text{ has } n \text{ distinct zeros } x_k \in [-1, 1] : x_k = \cos\left(\frac{(2k+1)\pi}{2n}\right) \text{ for } 0 \leq k \leq n-1.$$

2-2. Show that the Chebyshev polynomial T_n is a solution of the differential equation: [8]

$$(1 - x^2)\frac{d^2 f}{dx^2} - x\frac{df}{dx} + n^2 f = 0.$$

2-3. Use the identity/formula: $\displaystyle\sum_{k=0}^{N} \cos(\varphi + k\alpha) = \frac{\sin\frac{(N+1)\alpha}{2}\cos(\varphi + \frac{N}{2}\alpha)}{\sin\frac{\alpha}{2}}$

to show that: [12]

$$\sum_{k=0}^{N} T_m(x_k)T_n(x_k) = 0, \quad \text{for } m \neq n,$$

where $x_k = \cos\left[\dfrac{(2k+1)\pi}{2(N+1)}\right]$, $0 \leq k \leq N$, are the roots of T_{N+1}.

Problem 3 [45 Marks]

3-1. Given the integral

$$\int_0^3 \frac{\sin(2x)}{1+x^5} dx = 0.6717578646\cdots$$

3-1-1. Compute $T(J) = R(J, 0)$ for $J = 0, 1, 2, 3$ using the sequential trapezoidal rule. [10]

3-1-2. Use the results in **3-1-1.** and Romberg's rule to compute the values for the sequential Simpson rule $\{R(J, 1)\}$, sequential Boole rule $\{R(J, 2)\}$ and the third impprovement $\{R(J, 3)\}$. Display your results in a tabular form. [12]

3-2. State the three-point Gaussian Rule for a continuous function f on the interval $[-1, 1]$ and show that the rule is exact for $f(x) = 5x^4$. [5]

3-3. Use Jacobi's method to find the eigenpairs of the matrix

$$A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 5 & -2 \\ 4 & -2 & 1 \end{bmatrix}$$

[18]

God bless you !!!